កាលិកបត្រស្រាវជ្រាវមនុស្សសាស្ត្រនិងវិទ្យាសាស្ត្រសង្គម

Cambodian Journal of Humanities and Social Sciences

Website: https://cjhss-journal.com/

Analysis of Cambodian Students' Errors in Solving Quadratic Function Problems Using Newman's Error Analysis

ការវិភាគកំហុសក្នុងការដោះស្រាយចំណោទគណិតវិទ្យាទាក់ទងនឹងអនុគមន៍ដឺក្រេទី២ដោយប្រើវិធីវិភាគកំហុសរបស់ញូមែន

Tong Ly[1], Takuya Baba[2]

[1] Institute of Humanities and Social Science, Royal Academy of Cambodia
Email: lytongcambodia2013@gmail.com

[2] Professor of Mathematics Education at Hiroshima University, Japan

Received: May 12, 2023; Accepted: June 12, 2023; Published: July 10, 2023

CORRESPONDENCE: lytongcambodia2013@gmail.com

Citation: Ly, T., & Takuya, B. (2023). Analysis of Cambodian Students' Errors in Solving Quadratic Function Problems Using Newman's Error Analysis. *Cambodian Journal of Humanities and Social Sciences*, *2*(1), 32-44.

ABSTRACT

Errors and misconception in students' mathematics learning have become hot topics for teachers, educators and researchers for decades. Consistent errors may lead to misconception which ultimately affect the students' learning performance. This study centered on errors made by the students when they attempted to solve mathematics word problems. The objective of this study was to identify the errors the students made in solving quadratic function word problems. Qualitative method with students' test paper analysis and the in-depth interview using Newman's Error Analysis (NEA) framework were used to identify the students' errors and mistakes. Forty grade 12 students studying in a Cambodian public high school participated in the study. It was found that the students made the most errors in comprehension level (36.2%), reading errors accounted for 2.15% while transformation errors made up of 12.27%. Process skills errors made up of 19.94% while encoding errors (29.44%) ranked second among errors the students made. Failure to know or understand individual mathematical concept, for example vertex, symmetry line, interception point, range and domain of function etc. had hindered the students' comprehension of the problem and led to difficulties in solving math problems. This study suggested careful attention should be taken into account when introducing specific mathematics concept to students, especially at the early stage. Emphasizing the relationship between each concept helped strengthen the students' comprehension of the problem and improve their overall mathematics learning ability.

KEYWORDS: mathematics word problem, error, Newman's Error Analysis, quadratic function, Cambodian student

សង្ខិត្តន័យ

ការសិក្សាអំពីកំហុស និងការយល់ខុសទាក់ទងនឹងការរៀនមុខវិជ្ជាគណិតវិទ្យារបស់សិស្សបានក្លាយជាប្រធានបទដ៏សំខាន់មួយសម្រាប់គ្រូបង្រៀន អ្នកអប់រំ និងអ្នកស្រាវជ្រាវទាំងឡាយ។ កំហុសដែលសិស្សបានបង្កើតឡើងច្រំដែលៗអាចក្លាយទៅជាការយល់ខុស និងចុងក្រោយប៉ះពាល់ដល់លទ្ធផលនៃការសិក្សាលើមុខវិជ្ជាគណិតវិទ្យារបស់ពួកគេ។ ការសិក្សានេះផ្តោតជាចម្បងទៅលើកំហុសដែលសិស្សអាចនឹងបង្កើតឡើង នៅពេលពួកគេព្យាយាមដោះស្រាយចំណោទគណិតវិទ្យា។ វត្ថុបំណងនៃការស្រាវជ្រាវគឺសិក្សានិងកំណត់នូវប្រភេទកំហុសដែលសិស្សបង្កើតឡើងក្នុងការដោះស្រាយចំណោទគណិតវិទ្យាទាក់ទងនឹងអនុគមន៍ដឺក្រេទី២។ ការសិក្សាប្រើប្រាស់វិធីសាស្ត្រស្រាវជ្រាវបែបគុណភាព ជាមួយនឹងការវិភាគទៅលើសន្លឹកកិច្ចការប្រឡងរបស់សិស្ស បូកនឹងការសម្ភាសបែបស៊ីជម្រៅដោយវិធីវិភាគកំហុសរបស់ញូមែន (Newman's Error Analysis[1])។ សិស្សានុសិស្សថ្នាក់ទី១២ចំនួន៤០នាក់ដែលកំពុងសិក្សានៅវិទ្យាល័យរដ្ឋមួយក្នុងប្រទេសកម្ពុជាបានចូលរួមក្នុងការសិក្សានេះ។ ការស្រាវជ្រាវជ្រាវបានរកឃើញថា ក្នុងអំឡុងពេលដោះស្រាយចំណោទអនុគមន៍ដឺក្រេទី២ សិស្សទាំងឡាយបានបង្កើតនូវកំហុសច្រើនបំផុតក្នុងកម្រិតយល់ដឹង (36.2%), កំហុសក្នុងការអានមានចំនួន 2.15% ខណៈកំហុសក្នុងការកំណត់វិធីដោះស្រាយមានចំនួន 12.27%។ កំហុសក្នុងដំណើរការដោះស្រាយចំណោទមានចំនួន 19.94% ស្របពេលដែលកំហុសក្នុងការទាញបញ្ជាក់លទ្ធផលមាន 29.44% និងស្ថិតក្នុងចំណាត់ថ្នាក់ទីពីរច្រើនជាងគេ។ កង្វះខាតការដឹងឬយល់ពីបញ្ញត្តគណិតវិទ្យានីមួយៗ ឧទាហរណ៍ដូចជា ចំណុចកំពូលនៃខ្សែកោង បន្ទាត់ឆ្លុះ ចំណុចប្រសព្វរវាងខ្សែកោងនឹងអ័ក្សកូអរដោនេ ដែនកំណត់ និងសំណុំរូបភាពនៃអនុគមន៍ជាដើម បានរារាំងដល់ការយល់ដឹងរបស់សិស្សអំពីចំណោទបញ្ហាទាំងមូល ហើយបានក្លាយជាឧបសគ្គសម្រាប់សិស្សក្នុងការដោះស្រាយចំណោទគណិតវិទ្យា។ ការសិក្សានេះអាចក្លាយទៅជាសារមួយដ៏សំខាន់សម្រាប់គ្រូបង្រៀន ឱ្យបង្កើនការយកចិត្តទុកដាក់ឱ្យបានខ្ពស់ក្នុងការបង្រៀននិងពន្យល់នូវខ្លឹមសារបញ្ញត្តគណិតវិទ្យាជាក់លាក់ណាមួយដល់សិស្ស។ ការពន្យល់ពីទំនាក់ទំនងរវាងបញ្ញត្តគោលនីមួយៗអាចជួយពង្រឹងការយល់ដឹងរបស់សិស្សអំពីលំហាត់ឬចំណោទគណិតវិទ្យា និងបង្កើនសមត្ថភាពក្នុងការរៀនរបស់សិស្សលើមុខវិជ្ជានេះផងដែរ។

ពាក្យគន្លឹះ: ចំណោទគណិតវិទ្យា, កំហុស, វិធីវិភាគកំហុសរបស់ញូមែន, អនុគមន៍ដឺក្រេទី២, សិស្សានុសិស្សកម្ពុជា

1. INTRODUCTION

In general education[2], mathematics plays very important role in strengthening students' critical thinking and problem-solving skills etc. Mathematics has been regarded as one of the most important subjects among other school subjects. In relation to this, Jha (2012) and Peter (2011) mentioned that mathematics plays major role in improving the person's mind, influencing his/her reasoning ability and developing the person's personality.

In any country, mathematics and the national language are two most compulsory subjects to be taught at every school level. The requirement of mathematics increases throughout grade levels due to its role in scientific, technology and human development. Peter (2011) stressed that mathematics

[1] It should be note that the Khmer terms for Newman's five errors have not officially defined, but in this paper the researchers used tentative translation as follow: reading error-កំហុសក្នុងការអាន, comprehension error-កំហុសក្នុងការយល់, transformation error-កំហុសក្នុងការកំណត់វិធីដោះស្រាយ, process skills error-កំហុសក្នុងដំណើរការដោះស្រាយ and encoding error-កំហុសក្នុងការទាញបញ្ជាក់លទ្ធផល.

[2] In Cambodia, the term general education refers to the education system comprises of 12 grades: primary level, lower secondary level and upper secondary level.

is considered to be the only subject which is universally used in all cultures of the world to produce the educated man. Despite its importance, the learning of mathematics are still major issues for learners at all levels.

1.1. Research Problem

In mathematics classroom, two types of mathematical problems are primarily focused: (i) ready-to-solve math problem and (ii) mathematics word problem. Solving mathematics word problem is one of an integral part of the students' learning. In ready-to-solve problem, the students are not required to think about how the equation or expression of problem is generated or what phenomena an equation is represented; however, in math word problem, the students need to critically analyze the real-word phenomena, draw symbol, expression, equation or mathematics representation before proceeding to the calculation process.

Table 1
Interview *questions followed the Newman's Error Analysis prompts*

Problem	Example
Ready-to-solve math problem	Calculate the root(s) of the equation $x^2 + 2x + 3 = 0$ in real number domain.
Mathematical word problem	The sum of two numbers is 18, and the product of these two numbers is 56. What are the numbers?

According to Seifi et al. (2012), dealing with real-world problems that requires critical thinking, process skills solutions, pictorial and symbolic representation and interpretation is even more important than just as equations or math expression ready to be solved. For the first example in Table 1 above, the student can proceed to solve the quadratic equation directly after understanding the problem. They can use whatever method they know to solve the problem. However, for the second example, the students need to go through several steps, for example:

- First, read and try to understand the problem's requirement.
- Determine the correct mathematical expression or equation representing the math situation.
- Determine the best way to solve the problem.
- Proceed with the calculation process.
- Provide written solution to the problem and fit the math situation.

Due to the nature, math word problem poses more difficulties to learners compared to the ready-to-solve math equations. The issues in learners' difficulties in solving mathematics word problems have attracted attention among teachers and mathematics educators alike (Capone et al., 2021; Fatmanissa & Novianti, 2022; Seifi et al., 2012). Despise of the presence of the students' difficulties in mathematics learning, however, the study focusing on this issue is not much popular in Cambodia's context.

There were several studies conducted by MoEYS and partners as well as independent researchers; however, those existing literature focused on general issues in education sector, for example issues in research and higher education in Cambodia (Eam, 2015, 2016; Un, 2022; Williams et al., 2016), issues in English learning or instruction (Heng, 2012; Chhom & Kep, 2022; Em, 2022; Phon, 2017; Seng & Tep, 2022), issues related to student's learning achievement (Heng, 2013; Soeung et al., 2012), issues on school dropout (Heng et al., 2016) and issues about teachers and teaching profession (No & Heng, 2017). There were some studies about mathematics education in Cambodia (Khieu et al., 2019; Ly et al., 2022; Mong, 2020); however, those studies covered general issues in mathematics teaching and learning, but none focused on issues in students' mathematics learning with specific math contents.

Lack of knowledge in the issue, scarcity of scientific findings and lack of attention and consideration in mathematics education research motivates this study. The present study intended to fill the gap in scientific knowledge about contemporary issues in Cambodian mathematics education, especially focusing on a specific topic: quadratic function.

1.2. Research Objectives

The current research focused on the students' errors in solving quadratic function problems. Specifically, the objective of the study was to identify the students' errors in solving quadratic function word problems using Newman's Error Analysis (NEA) framework. The study was guided by the following question:

At which level, according to Newman's Error Analysis framework, do the students commit error/s in their attempt to solve quadratic function word mathematics?

2. LITERATURE REVIEW

2.1. Quadratic Function Concepts

In mathematics education, function concepts play very important roles in mathematics learning. Function concepts have been explicitly stated at higher grades; however, the natural concepts of function buried in all elements of mathematics from the early grades of study to the advanced level of mathematics learning. At primary school education, function concepts are implicitly introduced in mathematics curriculum in the form of concrete topics such as percentage, fraction, proportion etc. The concepts are later explicitly built from primary concepts of percentage, fraction and proportion into linear function and more complicated concepts throughout the grades. Other concepts of function such as quadratic, trigonometric, exponential and logarithm functions etc. are studied in higher grades following linear and non-linear function concepts.

In Cambodian mathematics curriculum, the concepts of quadratic function are explicitly introduced in grade 8 covering topics such as creating the table of values, identifying the domain and range (image) of the function, solving quadratic equation using different methods, for example, discriminant method, perfect squared method, factorization method etc. The concepts of solving quadratic inequalities, vertex, axis of symmetry, interception points, tangent line, common form ($y = f(x) = ax^2 + bx + c$) and its standard vertex form $y = f(x) = a(x - h)^2 + k$ are also introduced step by step throughout higher grades. Moreover, more complicated concepts such as the relationship between the graph and its coefficients, the roots and its discriminant ($\Delta = b^2 - 4ac$), etc. are covered. These concepts are then used extensively in limits of functions, derivatives and integration in senior secondary mathematics education and university mathematics education.

2.2. Students' Errors in Mathematics

Error is very common in the process of learning and solving mathematical problems, and sometimes students make certain mistakes during the process of their calculation. Errors or mistakes can be valuable sources for teachers to understand students' way of thinking because when solving mathematical problems, students usually put their knowledge and understanding into the calculation process (Cahyani et al., 2021). Error patterns in mathematics calculation reveal misconceptions about what they have learned, and that misconceptions are derived from problems due to conceptual misunderstandings (Ashlock, 2006; Holmes et al., 2013; Muzangwa & Chifamba, 2012). These conceptual misconception or misunderstanding may affect the students' future learning in mathematics. Teacher needs to be aware of students' mistakes, especially the persistent errors that might contain students' misconceptions about the topic they are learning.

Errors are usually divided into two types namely systematic error and unsystematic error (random error). An error can be a mistake, miscalculation or misjudge and such category falls under unsystematic errors. According to Muzangwa & Chifamba (2012), misrepresentation of a mathematics concept is not a misunderstanding that may produce a misconception in learning. Systematic error fall into the consistent errors or mistakes students made which caused by their conceptual misunderstanding. According to Holmes et al. (2013) and Muzangwa & Chifamba (2012), misconceptions are derived from problems due to conceptual misunderstandings and mistakes are derived from computational or minor mishaps or carelessness.

2.3. Newman's Error Analysis

Newman Error Analysis (NEA) is an error identification framework proposed by a famous Australian mathematics educator, Anne Newman, in 1977. The model has been proved to be a reliable tool for studying students' errors in mathematical word problems (Muzangwa & Chifamba, 2012; Prakitipong & Nakamura, 2006; Singh et al., 2010; White, 2010; Zakaria et al., 2010). The method had also been used in science subject research, for example Haqq et al. (2021). During the mathematical problem-solving processes, the students will commit errors and, according to Newman, these errors may be categorized into the following stages:

Reading ➔ Comprehension ➔ Transformation ➔ Process Skills ➔ Encoding

While teaching mathematics and when encountered with students who struggle in learning this subject, teachers usually said that their students could not solve mathematics problems correctly and that they made errors in mathematics problem solving. Newman, however, suggested that the students may commit errors in reading or comprehension of the problem before they arrive at pure mathematics problem-solution process. For Newman's method, it is assumed that in the process of problem solving, there are two kinds of obstacles that hinder students from arriving at correct answers (Jha, 2012, p. 17; Prakitipong & Nakamura, 2006, p. 113).

- *Troubles in reading language and abstract understanding of the problem which hinder their understanding the meaning of problems.*
- *Troubles in processing the mathematical problems that consists of transformation, process skills, and encoding answers.*

According to Newman, in order to assess the students' errors, teachers need to ask the correct questions so that these errors can be detected from their mathematical thinking which also gave the birth to her proposed error analysis framework.

- *Reading Error*

  Reading error occurs when the student fail or cannot recognize the written words, symbols or mathematical expressions in the given problem.

- *Comprehension Error*

  Comprehension errors occurs when the student could read or recognize the words, expression or mathematics symbols properly but they failed to understand its requirement or fully comprehend the problem, thus causing him/her to make mistakes or unable to get to problem-solution process.

- *Transformation Error*

  Transformation error occurs when the student has correctly comprehended the question or the requirement in the mathematics problem but they failed to identify proper mathematical operation or sequence of operations to successfully pursue the course of problem-solution.

- *Process Skills Error*

  Process skill error occurs when the student could correctly identify the proper procedures (or sequence of operations) for the problem, but they failed to execute or carry out the mathematical operation or procedures correctly.

- *Encoding Level*

  Encoding error occurs when, despite having solved the mathematical tasks appropriately and correctly, the student fails to provide an acceptable written form of the answer.

The following table presents some prompts which can be used to detect the student's error in specific stage, according to Newman.

Table 2
Interview questions followed the Newman's Error Analysis prompts

Questions	Error Stages
1. Can you read the question to me? 2. Do you understand the problem?	Reading
3. How confident are you that you can do this problem? 4. Tell me what is the question asking you to do?	Comprehension
5. Tell me the method you can use to find an answer to the question	Transformation
6. Show me how you worked out the answer to the question 7. Can you show me the working steps that you have used in order to find the answer? 8. Explain to me what you are doing as you do it 9. Do you know you are right? Why?	Process Skills

10. Can write down your answer to the question? 11. Tell me what is your answer? 12. How confident do you feel about the answer?	Encoding

3. METHODOLOGY

This study employed a qualitative-based research methodology where quadratic function knowledge test and in-depth interview using Newman's Error Analysis prompts were used as data gathering methods. A total of forty grade 12 students studying in a Cambodian public high school participated in this study.

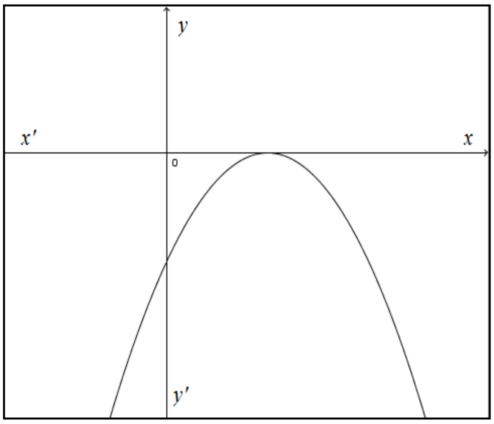
3.1. Instrument and Data

- **Quadratic Word Problem Test**

The quadratic word problem test consisted of six items covering relevant concepts in quadratic function. The test was designed to capture the characteristics of the students' knowledge and problem-solving skills in quadratic function concepts. The testing concepts for each item were presented in the following table.

Table 3

Quadratic function concepts measured in each item of the instrument

<table>
<tr><th>Item</th><th>Measuring mathematical concepts</th></tr>
<tr><td>1</td><td>Given $y = f(x) = -x^2 - 2x + 3$
(a) Write the equation of this function into standard form $y = f(x) = a(x-h)^2 + k$.
(b) Find the coordinates of vertex, axis of symmetry, and y-interception of the graph of this function.</td></tr>
<tr><td>2</td><td>(a) Please explain how you get the graph of the function $f(x) = x^2 + 2x - 1$ from the graph of $y = x^2$ then plot the graphs of these two functions on the same plane.
(b) Given $-4 \leq x \leq 6$ for the quadratic function $y = f(x) = x^2 + 2x - 1$. Find the domain and range of this function.</td></tr>
<tr><td>3</td><td>Determine the value of m for which the equation $x^2 + (m+1)x + 1 = 0$ has two different real roots.</td></tr>
<tr><td>4</td><td>Make the table values for $y = -\frac{1}{2}x^2$ and $y = x^2$ then plot the graphs of these two functions. Compare the graphs of $y = x^2$ and $y = -\frac{1}{2}x^2$.</td></tr>
<tr><td>5</td><td>Suppose that the profit or loss of a factory depends on a function $f(t) = t^2 - 2t - 3$, where t is the amount of products the company sells. For which value of t that the factory doesn't have any profit or loss?</td></tr>
<tr><td>6</td><td>Given the graph of a quadratic function
$y = f(x) = ax^2 + bx + c, \quad a \neq 0$


Explain the relationship between the graph of this function and its coefficients a, b and c and discriminant (Δ).
<table>
<tr><th>Expression</th><th>Positive</th><th>Negative</th><th>Zero</th><th>Why</th></tr>
<tr><td>a</td><td></td><td></td><td></td><td></td></tr>
<tr><td>b</td><td></td><td></td><td></td><td></td></tr>
<tr><td>c</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Δ</td><td></td><td></td><td></td><td></td></tr>
</table></td></tr>
</table>

- **In-depth Interview**

Face-to-face in-depth interviews took place one day after the students completed the test, and the interview lasted from 25 minutes to 30 minutes. The interview questions followed the Newman's Error Analysis prompts presented in Table 1 above. There were two practical ways of interview could be used as illustrated below:

(i) during the interview process on a specific item, if the researcher found that the students made an error at any stage of Newman's error, the interview was stopped.

(ii) during the interview process on a specific item, even any stage of Newman's error was detected, the interview still proceeded until the end of that item.

For this study, the researchers opted for the second scenario due to its benefit in capturing detailed information of knowledge and problem-solving skills the students' processed.

3.2. Data Analysis

The data analysis was made based on the solution papers submitted from the students and the interview scripts conducted with the participating students. The interview contents were transcribed and analyzed using content analysis based on Newman's Error Analysis framework. The errors the students made were categorized into (i) reading error, (ii) comprehension error, (iii) transformation error, (iv) process skills and (v) encoding error.

4. RESEARCH FINDINGS

There were forty students participated in this study. The age of the students ranged from 17 to 19 years old during the study was conducted.

4.1. Students' Errors According to Newman's Framework

It was revealed that out of the six testing items, the forty participating students made a total of 326 errors, according to Newman's Error Analysis framework. Not all the six testing items were solved by each student, and some problems were left blank. In this case, the blank problems were left out of the analysis.

Table 4

The distribution of the students' errors according to Newman's Error Analysis

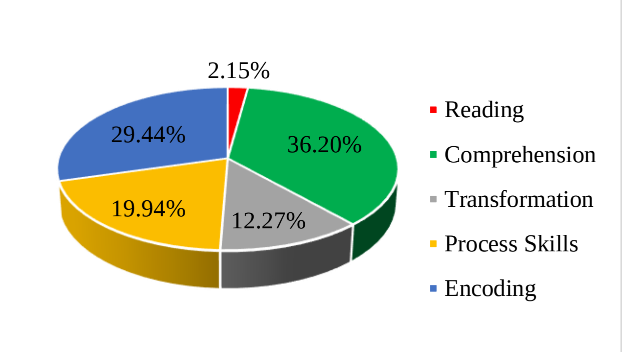
Student	Reading	Comprehension	Transformation	Process Skills	Encoding	Total
S1	0	3	0	2	3	9
S2	0	5	1	1	3	10
S3	0	3	0	2	2	7
S4	0	4	0	2	3	9
S5	0	4	1	2	5	13
S6	0	5	1	0	1	7
S7	0	5	6	5	6	22
S8	0	4	4	4	5	17
S9	0	5	5	4	5	19
S10	0	5	2	3	4	14
S11	2	3	3	4	4	16
S12	0	5	2	4	5	16
S13	0	4	2	4	6	16
S14	0	5	0	1	2	8
S15	0	5	3	5	5	18
S16	0	4	2	5	5	16
S17	0	4	0	2	4	10
S18	0	4	1	3	3	11
S19	1	5	3	3	4	16
S20	2	5	4	4	6	21
S21	1	1	0	0	1	3
S22	1	2	0	0	0	3
S23	0	2	0	1	1	4
S24	0	1	0	1	1	4
S25	0	2	0	0	0	2
S26	0	2	0	1	1	4
S27	0	2	0	1	1	4
S28	0	2	0	0	0	2
S29	0	1	0	0	1	2
S30	0	1	0	0	0	1
S31	0	2	0	0	1	3
S32	0	1	0	0	1	2
S33	0	2	0	0	1	3
S34	0	2	0	0	0	3
S35	0	2	0	1	1	4
S36	0	2	0	0	1	3
S37	0	1	0	0	1	2
S38	0	1	0	0	1	2
S39	0	1	0	0	1	3
S40	0	1	0	0	1	2
Total	*7*	*118*	*40*	*65*	*96*	*326*

The students made the most errors at comprehension level, i.e., comprehension error, which accounted for 36.2% of all the errors produced. In addition, the students committed several process skills errors (19.94%), despise having identified the correct ways to solve the problems. Surprisingly, encoding errors accounted for considerably high proportion which made up to 29.44% of the total errors made. Despise having solved the problem correctly, the students still made mistakes in encoding level, that is they failed to provide the proper solutions to the questions asked in wording or written formats.

The study also revealed that reading errors were related to the students' misreading of mathematics operations, expressions and specific mathematics vocabularies etc., however, these mistakes were not found to be obstacles for the students' comprehension of the problems.

Figure 1

Distribution of the students' errors based on Newman's Error Analysis



It was evidenced that the students had difficulties in conceptualizing individual mathematical concepts, for example, vertex, domain of function and range of function etc. The students' poor conceptualization in the mathematical concepts had, obviously, hindered their understanding about the problems and failed in the subsequent steps of problem-solving processes.

The transformation errors were strongly related to how much the students could comprehend and understood the problems' requirement; however, some students even though they could identify the problems' requirement, they still committed errors in transformation level due to their poor memorization of the correct mathematical formulas.

4.2. Examples of the Analysis of the Student's Errors Based on Newman's Framework

This section presented some examples of the analysis of the errors the students committed when they tried to solve the quadratic function word problems in the test paper. The errors were classified based on Newman's Error Analysis framework: (i) reading error, (ii) comprehension error, (iii) transformation error, (iv) process skills error, and (v) encoding error.

➲ Reading Errors

A reading error occurred when the students could not recognize the written words, symbols or math equation or expressions. Misreading of the words, symbols or those expressions also falls into this error category.

Problem 2-B:

A. Please explain how you get the graph of the function $f(x) = x^2 + 2x - 1$ from the graph of $y = x^2$ then plot the graphs of these two functions on the same plane.

B. Given $-4 \leq x \leq 6$ for the quadratic function $y = f(x) = x^2 + 2x - 1$. Find the domain and range of this function.

(R: Researcher, S8: Student Number 8)

R: Can you read problem 2-B to me?
S8: Given -4 *greater than or equal to* x *greater than or equal to* 6*, for the quadratic function* $f(x) = x^2 + 2x - 1$*. Find the domain and range of this function.*
R: After reading, did you have any word or expression that you were not familiar with?
S8: I didn't know many things in the question. I didn't know what is the domain or range of function.

The student failed to read the symbol " $\leq$ " correctly in the given expression $-4 \leq x \leq 6$, instead of reading it as *"-4 is less than or equal to x and less than or equal to 6"*, the student read it as *"-4 is greater than or equal to x and greater than or equal to 6"*.

➲ Comprehension Error

A comprehension errors occurred when the students were able to read the question or were able to recognize the mathematics symbols properly, but they failed to understand the problem's requirement, thus causing him/her to make mistakes, could not fully understand the problem and was unable to get to problem-solution.

Problem 1-A:

Given the function $y = f(x) = -x^2 - 2x + 3$

A. Write the equation of this function into standard form $y = f(x) = a(x - h)^2 + k$.

B. Find the coordinates of vertex, axis of symmetry, and *y*-interception of the graph of this function.

(R: Researcher, S13: Student Number 13)

R: Can you read problem 1-A to me?
S13: [Reading]…
R: After reading, did you see any unfamiliar word or expression?
S13: I didn't know the standard form, teacher.
R: Did you mean the word 'standard form?'
S13: Yes teacher.

R: How about the math expression $y = f(x) = a(x-h)^2 + k$ here? Have you ever seen it before?
S13: I used to see it before.
R: Overall, did you understand the question asked?
S13: I understood, but I didn't know what exactly is to transform it into the standard form.
R: Did you think you can do it or not?
S13: No, teacher. Actually, I didn't know which standard form I need to transform to as I didn't know anything about it.

The student mentioned that she understood the problem, but her words "*I understood, but I didn't know what exactly to transform to the standard form*" indicated that she did not understand the requirement of the problem and what it meant by the standard vertex form, which was comprehension error.

➲ Transformation Error

A transformation error occurred when the pupil could correctly comprehend the problem or the question's requirement but failed to identify the proper math formula, operation or sequence of operation to successfully pursue the course of problem-solution process.

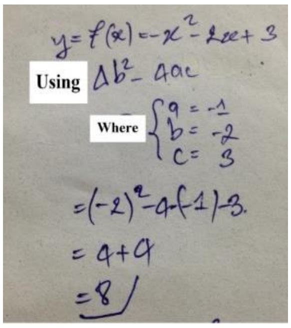
Problem 1-A:

Given the function $y = f(x) = -x^2 - 2x + 3$

A. Write the equation of this function into standard form $y = f(x) = a(x-h)^2 + k$.
B. Find the coordinates of vertex, axis of symmetry, and *y*-interception of the graph of this function.

(R: Researcher, S10: Student 10)

R: Can you read problem 1-A to me?
S10: [Reading]
[...]
R: I saw you did some calculation on your paper; can you tell me why you chose to solve this problem using the delta method?



S10: For this case, $\Delta = b^2 - 4ac$. I chose it because I looked at the given expression $y = f(x) = -x^2 - 2x + 3$ at which the values of $a = -1$, $b = -2$ and $c = 3$.
R: Could you tell more clearly why you chose delta method ($\Delta = b^2 - 4ac$) to solve this problem?
S10: I could not find out other way for this problem, so I chose to do it using this method.

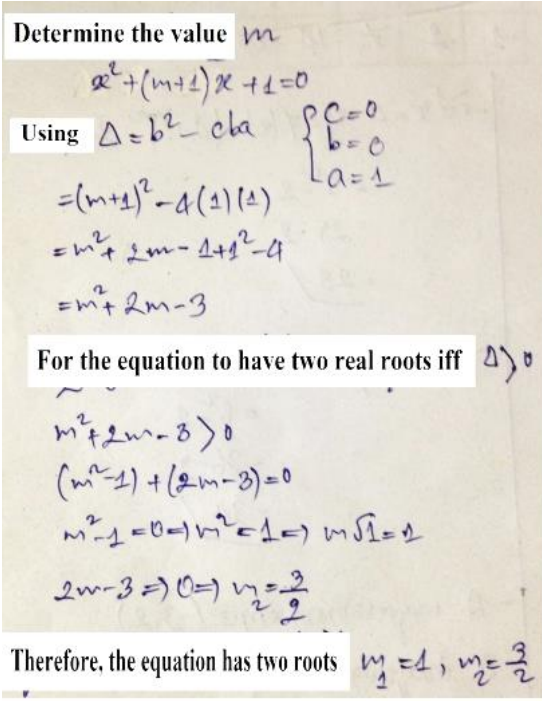
From the interview, the students could understand the problem requirement, but they chose an incorrect way to solve it. The students might have thought that the delta method could be used to transform the function $y = f(x) = -x^2 - 2x + 3$ into the standard vertex form ($y = f(x) = a(x-h)^2 + k$).

➲ Process Skills Error

A process skill error occurred when the pupil could correctly identify how to solve the problem or chose the correct formula or sequence of procedures to solve the problem, but they failed to carry out the procedures correctly.

Problem 3:

Determine the value of m for which the equation $x^2 + (m+1)x + 1 = 0$ has two different real roots.



(R: Researcher, S10: Student Number 10)

R: Can you read this problem to me?
S10: [Reading]
[...]
R: Did you know what is the question asking you to do?
S10: It asked to determine the value of m so that the equation had two different real roots.
R: OK, can you tell me how you get the problem done?
S10: Because the question asked to determine the value of m, so I took the given equation and solve it using the delta-method $\Delta = b^2 - cba$

where, for this case, $c = 0, b = 0$ and $a = 1$. Then I got: $\Delta = (m + 1)^2 - 4(1)(1)$ then I got $\Delta = m^2 + 2m - 3$. But we know that for the equation has two real roots if and only if $\Delta > 0$. Then I solved it one more time.

R: *Can I see what you are doing here? Let me ask you something more. You wrote $m^2 + 2m - 3 > 0$ and got $(m^2 - 1) + (2m - 3) = 0$, can you explain to me how did you get "-1" in the expression $(m^2 - 1)$?*

S10: *I got if from $2m - 3$*

From this interview, the student understood the problem's requirement that is to get the two different real roots. Moreover, he/she knew that the condition for this to happen when the value '$\Delta > 0$'. However, they chose the incorrect formula of delta method as well as committed a lot of errors during the process of problem solving. In this case, the student made mistakes in both transformation and process skills levels.

➲ Encoding Errors

An encoding error occurred when, despite having appropriately and correctly solved a mathematical task, the pupil failed to provide an acceptable written form of the answer to the problem. In this research, this error also included those mistakes made with the conclusion of the solutions.

<u>Problem 5:</u>

Suppose that the profit or loss of a factory depends on a function $f(t) = t^2 - 2t - 3$, where t is the amount of products the company sells. For which value of t the factory doesn't have any profit or loss?

(R: Researcher, S29: Student Number 29)

R: *Can you read this problem to me?*

[...]

R: *From your calculation, what is the answer for this problem?*

S29: *I got the values for t which are $t_1 = 3$ and $t_2 = -1$.*

R: *What do these values refer to?*

S29: *They are the amount of products that the factory needs to sell so that they will not have any loss or profit.*

R: *What is the conclusion for this problem?*

S29: *I concluded that, the factory needs to sell the products $t = 3$ & $t = -1$ so that they will not get any loss or profit.*

For this problem, many students made mistakes in drawing the written solution to the problem. They could provide the correct answer, but they lack the concept of amount which requires only positive value. For this problem, the students provided the solution with both positive and negative values of t.

5. DISCUSSION

It was found that the students made the most errors at comprehension level. The finding reflected the general characteristics of the students' difficulties in solving the mathematical word problems. Without restricting to a specific math content and research contexts, several past studies, for example Jha (2012), Prakitipong & Nakamura (2006), Singh et al. (2010), White (2010) and Zakaria et al. (2010), showed that during the process of solving mathematics problems many students made a lot of errors in comprehension before they arrived at problem-solving processes. Students struggled in understanding the problems' requirement which led to failing to identify the correct way to solve the problems. In addition to this, Jha (2012), Singh et al. (2010) and White (2010) added that that the students would make more errors in transformation level than in comprehension level although errors committed in comprehension level accounted for higher percentage comparing to the other levels. In relation to this, Jha (2012) found that , in Southeast Asia, comprehension and transformation errors accounted for about 70% of errors made by students on standard word problems.

From the study, reading errors were not found to be obstacles for the students' comprehension of the problems; however, lack of conceptual understanding did play major roles in hindering their comprehension. For example, in Problem 1-B, the students were asked to determine the coordinates of the vertex of the function $y = f(x) = -x^2 - 2x + 3$, the symmetric line and the y-interception point of the graph using the vertex form $y = f(x) = a(x - h)^2 + k$. From the interviews, however, many students did not know or they failed to understand the concept of '*vertex of graph of a function*'. Many students suggested that only the function with graph having maximum point (i.e., the quadratic graph with coefficient $a > 0$) should have vertex. The students may have confused the term '*vertex*' to the term used in Khmer language which is translated as the '*peak of something*'. In mathematics classes, many teachers introduced the term 'vertex' in Khmer as 'កំពូល' (*pronounced kompoul*) which was literally referred to the 'highest point' in Khmer language. The students may lack of a complete understanding of the concept of 'vertex' for a quadratic function that refer to 'the maximum point' or 'minimum point' depending on the value of the coefficient a.

Past experience of being a high school mathematics teacher in Cambodian public schools, the researcher argued that two major reasons might have contributed to the students' lack of conceptual understanding in each mathematical concept then led to their poor comprehension of the problems. The first reason may be the lack of focus on the concepts themselves during the teachers' introduction of the lessons. The second reason may be the habit of practicing mathematics problem solving. In Cambodian's mathematics classes, more practices are put in ready-to-solve mathematical problems rather than word problems. Failing to provide more practical exercises on the concepts, especially in word problems or real-world problem may hinder the students' holistic comprehension of the problems.

6. CONCLUSION

This study aimed at identifying errors the students made when they attempted to solve quadratic function word problems using Newman's Error Analysis framework. The study found that the students committed the most errors in comprehension level and encoding level. Reading errors were found to be related to misreading of mathematics signs, specific mathematics vocabularies etc., however, these mistakes were not found to be obstacles for students' comprehension of the problems. Failure to understand individual mathematical concept, for example vertex, symmetry line, interception point, range and domain of function etc., had hindered the students' comprehension of the problem were sources of the students' comprehension errors. More errors in encoding and lead to difficulties in solving the math problems. This study suggested careful attention should be taken into account when introducing specific mathematics concept to students. Emphasizing the relationship between each concept helped strengthen the students' comprehension of the problem and improve their overall mathematics learning ability.

INFORMATION

This paper is part of my unpublished thesis for the degree of master of mathematics education at Hiroshima University. Having realized the significant findings related to the issues in mathematics education research, the authors wished to share the findings to academic community by discussing with current relevant literature. The data was a bit old; however, the authors believed that the essence of the findings should still be relevant, especially in mathematics education in Cambodia. The study contributes primarily in pedagogical context where students' learning of mathematics is the main focus.

.

REFERENCES

Ashlock, R. B. (2006). *Error patterns in computation : using error patterns to improve instruction*.

Cahyani, N. P. I., Suarsana, I. M., & Mahayukti, G. A. (2021). Improving Student's Mathematical Problem-Solving Skills Through Relating-Experiencing-Applying-Cooperating-Transferring Learning Strategy and Graphic Organizer. *Proceedings of the First International Conference on Science, Technology, Engineering and Industrial Revolution (ICSTEIR 2020)*, *536*(Icsteir 2020), 337–344. https://doi.org/10.2991/assehr.k.210312.056

Capone, R., Filiberti, F., & Lemmo, A. (2021). Analyzing difficulties in arithmetic word problem solving: An epistemological case study in primary school. *Education Sciences*, *11*(10). https://doi.org/10.3390/educsci11100596

Chhom, C., & Kep, B. (2022). Students ' Engagement and English Performance : A Case of Undergraduate Students in the University of Cambodia 1. *Cambodian Journal of Humanities and Social Sciences*, *1*(April), 16–35.

Eam, P. (2015). Investigating Relationship among Research Self-Efficacy, Research Outcome Expectation, and Research Interest of

Cambodian Faculty: Testing Social-Cognitive Theory. *International Journal of Sociology of Education*, *4*(3), 199–224. https://doi.org/10.17583/rise.2015.1752

Eam, P. (2016). Factors Differentiating Research Involvement among Faculty Members: A Perspective from Cambodia. *Excellence in Higher Education*, *6*(1&2), 1–11. https://doi.org/10.5195/ehe.2015.133

Em, S. (2022). Challenges of English language learning and teaching in Cambodia: A case study of Kith Meng Brasat High School. *Cambodian Journal of Educational Research*, *2*(1), 62–80. https://www.researchgate.net/profile/Kimkong-Heng/publication/362688998_Cambodian_Journal_of_Educational_Research_Volume_2_Issue_1/links/62f8fdb352130a3cd71ed0aa/Cambodian-Journal-of-Educational-Research-Volume-2-Issue-1.pdf#page=69

Fatmanissa, N., & Novianti, M. N. R. (2022). Linguistic Challenges in Solving Mathematics Word Problems: A Case of EFL University Students. *Proceedings of the Eighth Southeast Asia Design Research (SEA-DR) & the Second Science, Technology, Education, Arts, Culture, and Humanity (STEACH) International Conference (SEADR-STEACH 2021)*, *627*, 16–23. https://doi.org/10.2991/assehr.k.211229.003

Haqq, W. M., Pratama, R. B., Mi, M., & Setiaji, B. (2021). *Investigating the Students ' Errors When Solving Analytical Mechanics U sing Newman ' s Error Analysis A . Introduction B . Method*. *1*(2), 63–71.

Heng, K. (2012). College Student Involvement in English Language Learning in Cambodia : Trends and Patterns. *Journal of International Development and Cooperation*, *19*(1), 13–23.

Heng, K. (2013). Factors Infulencing College Students' Academic Achievement in Cambodia: A Case Study. *Journal of Teaching and Learning in Higher Education*, *5*(2), 34–49.

Heng, K., Sok, S., & No, F. (2016). *School Dropout in Cambodia: A Case Study of Phnom Penh and Kampong Speu*.

Holmes, V., Miedema, C., & Haugen, N. (2013). Data-Driven Intervention: Correcting Mathematics Students' Misconceptions, Not Mistakes. *Mathematics Educator*, *23*(1), 24–44.

Jha, S. K. (2012). Mathematics performance of primary school students in Assam (India): An analysis using Newman Procedure. *International Journal of Computer Applications in …*, *II*(I), 17–21. http://search.ebscohost.com/login.aspx?direct=true&profile=ehost&scope=site&authtype=crawler&jrnl=22314946&AN=82881779&h=EVVQDh/Hm52chAMlwrzR4w4wi7jATMnDU2PdIoLeamUS/cc4YNQKyx+qkOW4Z0IJs51nz6+2xGbl75NrT1SSwg==&crl=c

Khieu, V., Om, S., Souk, S., & Khy, P. (2019). Mathematics Teaching: Improving Student Learning Acheivements in Mathematics in Grade Eight. *Cambodia Education Review*, *3*(1), 84–97.

Ly, T., Phauk, S., & Chea, S. (2022). Relationship between Students' Attitudes toward Mathematics and Their Achievement in Probabilities and Statistics: A Case of Engineering Students at ITC. *Cambodian Journal of Humanities and Social Sciences*, *1*(1), 36–53.

Mong, M. (2020). *Importance of Math and Difficulties in Learning Perceived by Cambodian Grade 10 and 11 students*. *5*(2), 52–57.

Muzangwa, J., & Chifamba, P. (2012). Analysis of Errors and Misconceptions in the Learning of Calculus by Undergraduate Students. *Acta Didactica Napocensia*, *5*(2), 1–10.

No, F., & Heng, K. (2017). Survey report on teachers and teaching profession in Cambodia. *Education Research Council, Cambodia*, *May*, 0–89.

Peter, E. E. (2011). Mathematics: Indispensable tool for successful and balance human existence on this planet. *Advances in Applied Science Research*, *2*(5), 449–456.

Phon, S. (2017). Factors affecting the English language proficiency of students majoring in English at a rural university in Cambodia. *UC Occasional Paper Series*, *1*(1), 69–92.

Prakitipong, N., & Nakamura, S. (2006). Analysis of mathematics performance of grade five students in Thailand using Newman procedure. *Journal of International Cooperation …*, *9*(1), 111–122. http://home.hiroshima-u.ac.jp/cice/wp-content/uploads/publications/Journal9-1/9-1-9.pdf

Seifi, M., Haghverdi, M., & Azizmohamadi, F. (2012). Recognition of Students' Difficulties in Solving Mathematical Word Problems from the Viewpoint of Teachers. *Journal of Basic and Applied Scientific Research*, *2*(3), 2923–2928. https://www.researchgate.net/profile/Majid-Haghverdi/publication/261548865_Recognition_of_Students'_Difficulties_in_Solving_Mathematical_Word_Problems_from_the_Viewpoint_of_Teachers/links/0f31753491034390c1000000/Recognition-of-Students-Difficulties-in-So

Seng, S., & Tep, S. (2022). Demystifying Perceptual Learning Style Preferences in English Academic Achievement : A Case Study of Finance and Banking Undergraduates in Cambodia. *Cambodian Journal of Humanities and Social Sciences*, *1*(April), 1–15.

Singh, P., Rahman, A. A., & Hoon, T. S. (2010). The Newman procedure for analyzing Primary Four pupils errors on written mathematical tasks: A Malaysian perspective. *Procedia - Social and Behavioral Sciences*, *8*(5), 264–271. https://doi.org/10.1016/j.sbspro.2010.12.036

Soeung, S., No, F., Ang, B., & Hirakawa, Y. (2012). *Factors Influencing Student Achievement in Rural Cambodian Primary Schools*. 1–17. http://www.chea.edu.kh/dl2014/soeung_sopha.pdf

Un, L. (2022). Higher Education and Its Research in Cambodia: For What and for Whom? *Nagoya Higher Education Research*, *23*.

White, A. L. (2010). Numeracy, literacy, and Newman's Error Analysis. *Journal of Science and Mathematics Education in …*, *33*(2), 129–148. http://www.recsam.edu.my/R%26D_Journals/YEAR2010/dec2010vol2/allan(129-148).pdf

Williams, J. H., Kitamura, Y., & Keng, C. S. (2016). Higher Education in Cambodia: Expansion and Quality Improvement. *International and Development Education*, 167–186. https://doi.org/10.1057/9781137456007_9

Zakaria, E., --, I., & Maat, S. M. (2010). Analysis of Students' Error in Learning of Quadratic Equations. *International Education Studies*, *3*(3), 105–110. https://doi.org/10.5539/ies.v3n3p105

Tong LY

Workplace:
- Royal Academy of Cambodia

Master's Degrees in:
- Mathematics Education (*Hiroshima University*)
- Educational Administration (*University of Cambodia*)
- Mathematics (*Royal University of Phnom Penh*)

Research Interest:
- Data Science
- Mathematics Education
- Student Learning and Misconceptions

(Unavailable)

Takuya Baba

Workplace:
Professor of Mathematics Education at Hiroshima University, Japan